Potts-like model for ghetto formation in multi-cultural societies

Christian Schulze

Institute for Theoretical Physics, Cologne University

D-50923 Köln, Euroland

February 2, 2008

e-mail: ab127@uni-koeln.de

Abstract: In a Potts-like model of Q ethnic groups, we follow Schelling (1971) and Meyer-Ortmanns (2003) and simulate the formation of ethnic ghettos as well as their prevention by an increasing social temperature.

Keywords: Sociophysics, Schelling model, phase separation, dynamics.

1 Introduction

Binary models like Ising-type simulations have a long history. They have been applied by Schelling [1] to describe the ghetto formation in the inner cities of the USA, i.e. to study phase separation between black and white. More recently, Meyer-Ortmanns simulated the Ising model with a temperature increasing with time and showed that in this way ghettos can be avoided: Higher temperature means higher tolerance towards different people [2, 3]. The present note aims to generalize this work to up to seven different ethnic groups, as may be more appropriate to European societies.

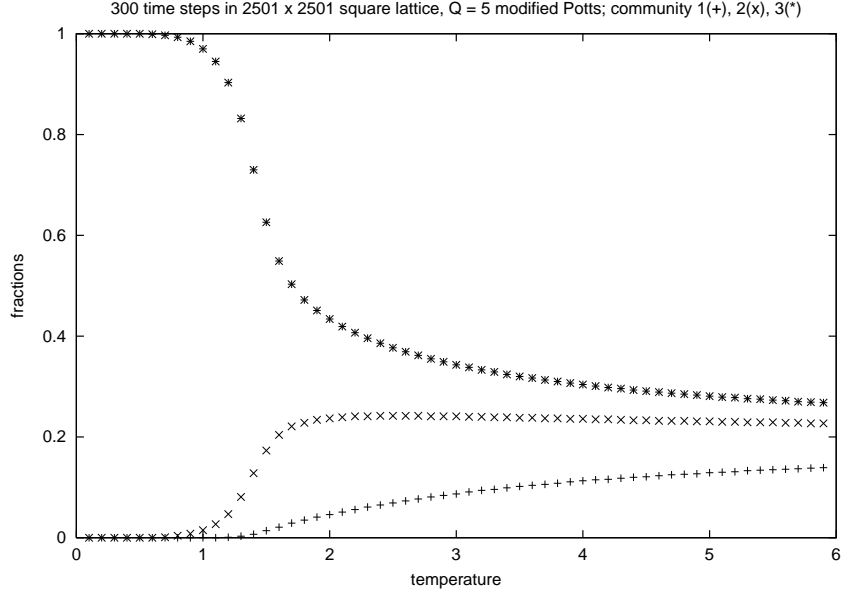300 time steps in 2501 x 2501 square lattice, Q = 5 modified Potts; community 1(+), 2(x), 3(*)

Figure 1: Equilibrium concentrations of five ethnic groups, starting with group 3 and allowing transformation of members of one group into members of another group.

2   Model

We assume the presence of Q different ethnic groups, numbered by an index q between 1 and Q. The similarities between q and $q \pm 1$ are taken as larger than those between groups with indices which are far apart. Thus, in contrast to the usual Potts model, we take the interaction energy between two groups i and k as proportional to the absolute value of the difference $q_i - q_k$:

$$E = J \sum |q_i - q_k| \tag{1}$$

where the sum goes over all nearest neighbours on a $L \times L$ square lattice, with helical bounday conditions. Different configurations are realized with a Boltzmann probability $\propto \exp(-E/k_BT)$ and the Boltzmann constant k_B; the temperature T thus is controlled by the dimensionless variable k_BT/J. However, we measure our social temperature in units of the critical Potts temperature $[2/\ln(1 + \sqrt{q})]J/k_B$. Thus, in the Ising case of $Q = 2$ the equilibrium critical temperature is $T = 1$ since with two groups the difference between our interaction energy and the usual Potts energy vanishes.
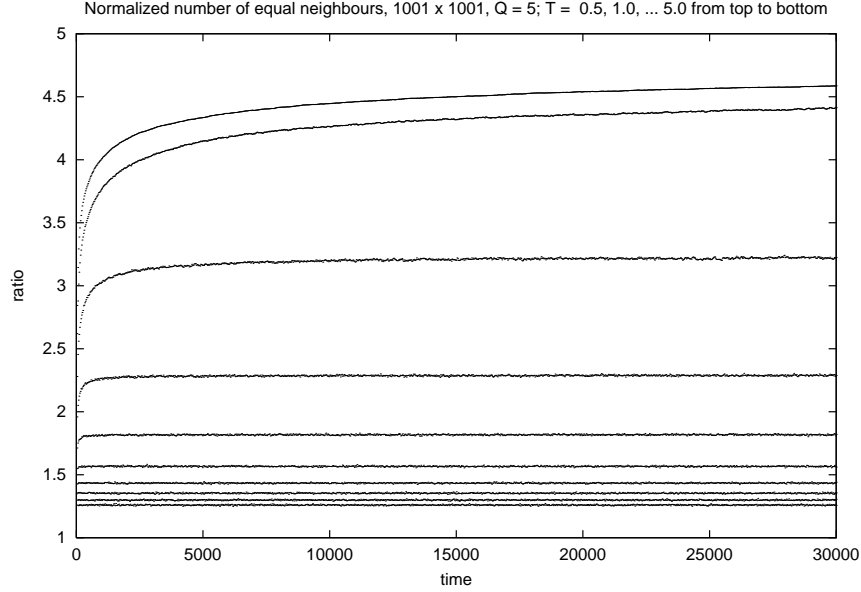
2

Figure 2: Time dependence of separation for five groups in one million lattice sites; the initial value (random distribution) is normalized to unity.

For equilibrium studies the details of the dynamics do not matter much and we use Glauber dynamics; for non-equilibrium instead we use Kawasaki dynamics with infinite-range exchange. Thus in the latter case each person randomly selects a site far from its own neighbourhood and tries to exchange its residence with the person on that site, according to the above Boltzmann probability.

3   Results

While the $Q = 2$ case (not shown) has a sharp phase transition at $T = 1$, with mixing for $T > 1$ and phase separation for $T < 1$, for higher Q (we simulated up seven cultures) no sharp phase transition exists. Fig.1 shows a typical case, $Q = 5$: At low temperatures, $q = 3$ dominates, at high temperatures, all q are roughly equally represented, without a sharp separation temperature in between. (Group 4 has about the same fraction as group 2, and group 5 agrees in size with group1.)
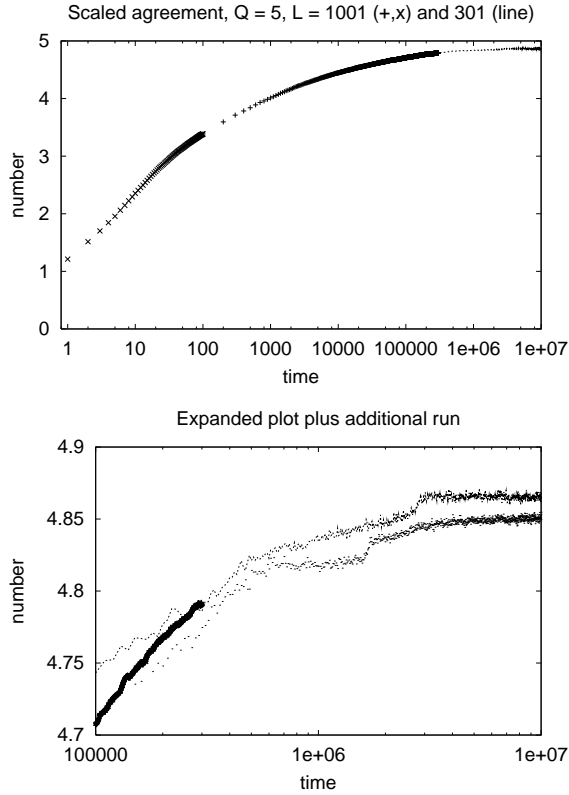
Figure 3: a: As Fig.2 but for longer times and $T = 0.5$ only. b: Same run as in part a, plus another run differing only by the random numbers, shown only for the later times.

For the dynamics, we no longer allow members to change the group to which they belong. To quantify the separation effects, we calculate the nearest-neighbour correlation function, i.e. the average number of equal neighbours which a site has. We normalize this number by its value in the initial configuration, where all groups are equally and randomly distributed among the lattice sites. This normalized number of equal neighbours is shown in Fig.2 for $Q = 5$ and various temperatures T: The higher is T, the lower is the separation as measured by the number of equal neighbours. Low temperatures show an unusual long-time dynamics, as shown in Fig.3 for $T = 0.5$.

Following Meyer-Ortmanns [2] we now assume that the tolerance towards other ethnic groups, as measured through the temperature T, increases with
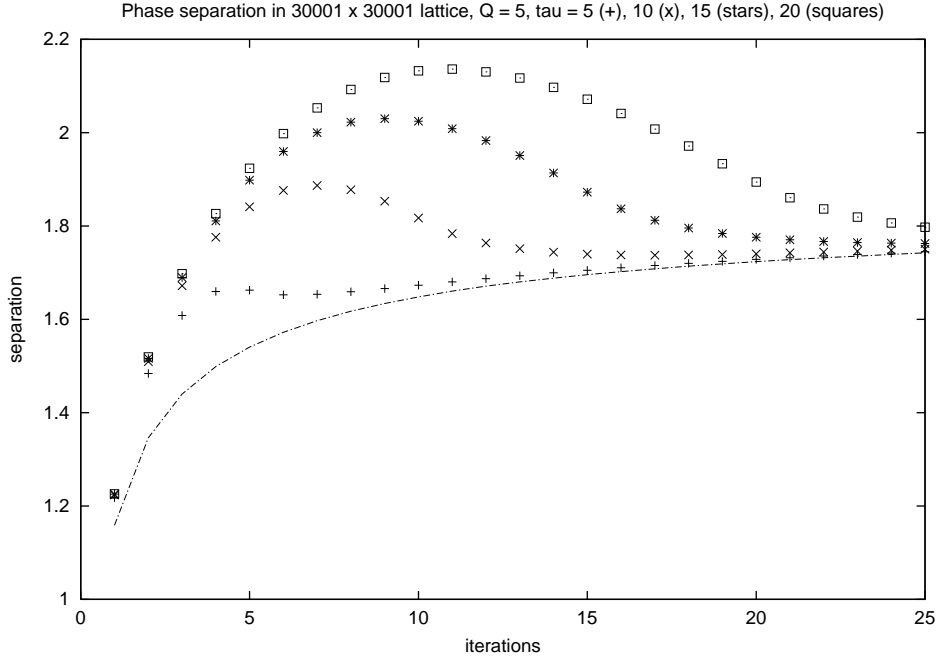
4

Figure 4: Time dependence of separation for five groups in 30001×30001 lattice with temperature increasing from 0.5 to 2.5 in the first τ iterations, with $\tau = 5, 10, 15, 20$ from bottom to top. The line shows the behaviour for fixed temperature $T = 2.5$ as in Fig.2. For 1001×1001 the deviations are of the order of the symbol size. The long-time value from Fig.2 is 1.82.

time. Thus we start with $T = 0.5$ which means according to Figs.2,3 a strong separation. Then, starting from the first iteration, we increase T linearly with time up to $T = 2.5$ (weak separation according to Fig.2) within τ iterations. We check if the resulting non-equilibrium separation (measured in the same units as Fig.2) remains below a threshold of two, i.e. is at most twice as high as for randomly distributed people. Fig.4 with 900 million people shows that for $\tau = 5$ the separation increases monotonically up to its equilibrium value near 1.8, for $\tau = 10$ it has a maximum below 1.9, for $\tau = 15$ the maximum is near 2.0, and for $\tau = 20$ it is clearly above 2. Thus tolerance has to increase fast enough if separation is to be avoided.

Thanks are due to D. Stauffer and M. Hohnisch for help and discussions.

References

[1] T.C. Schelling, J. Math. Sociol. 1, 143 (1971).

[2] H. Meyer-Ortmanns, Int. J. Mod. Phys. C 14, 311 (2003).

[3] W. Weidlich, Sociodynamics: A Systematic Approach to Mathematical Modelling in the Social Sciences. Harwood Academic Publishers, 2000.